Texto 1

DIVISION WITH REMAINDER IN UNIVERSITY AND IN PRIMARY SCHOOL: AN EPISTEMIC ANALYSIS

Cooper, J.; Karsenty, R.

In abstract algebra, division is a somewhat neglected operation. Its natural domain is field theory, where every nonzero element has a multiplicative inverse, and division is conceived as multiplication by inverse. In the more general structure of rings, division cannot be defined since not every element need have an inverse, yet in some cases (Euclidean rings), division with remainder (DWR) can be defined. The problem of dividing a by b is formulated as finding q and r, such that $r < b$ and $a = qb + r$. DWR is a more advanced topic than regular division and is typically taught in elective courses on advanced ring theory. A related topic is modulo arithmetic. Given a divisor b, $\mathbb{Z}$ can be separated into equivalence classes according to the remainder from division by b, and operations of addition and multiplication can be defined on these classes. In this context, the quotient (q) is of no significance, only the remainder is of consequence.

In elementary school, division is one of the four basic operations and has its typical word problem models. The most common models – partition and quotition – are first introduced in grade 1. The partitive model is described as "dividing into equal parts" (Pedagogical Secretariat of the Israeli Ministry of Education 2009, translated from p. 24), e.g., "12 pencils are divided into 3 boxes in such a manner that each box will hold the same number of pencils. How many pencils will there be in each box? (4 pencils)" (ibid., p. 25). The quotitive model is described as finding the number of parts of a given size, e.g., "12 pencils are divided into boxes in such a manner that each box will hold 3 pencils. How many boxes will we need? (4 boxes)" (ibid.). Of course, the dividend may not be divisible by the divisor, in which case there may be something left over. Division with remainder can be seen as an interim solution to the problem of defining and performing whole number division before students have been introduced (in grade 5) to "the meaning of fraction as the quotient of division" (ibid., p. 98). Accordingly, DWR nearly disappears from the school curriculum once fractions are learned. Indeed, if you ask educated adults to calculate $7 : 2$, they will probably answer *three and a half* or *three point five* (as obtained from a calculator), and not *three remainder one*.

In reality, our lives are filled with partitive and quotitive problems where items cannot be split. Such problems serve as models for DWR, even though many real-life situations are less strict about the range of solutions than abstract DWR: real life sharing need not always be perfectly fair, and parts need not always be of perfectly equal size. Thus, children are expected to engage in two different discourses of division. The abstract arithmetic discourse is about numbers – students calculate the result of a division exercise – whereas the discourse of models of division is about solving particular types of realistic problems. The discourse of models (e.g., fair sharing) shares some keywords with the arithmetic discourse, such as *divide* (a fair distribution of objects, but also an arithmetic operation on numbers), and *remainder* (a collection of objects left over but also a number).

In the arithmetic discourse, students calculate the result of division problems-quotient and remainder. The notation they use in Israel is, for example, $7 : 2 = 3(1)$. This may appear to be quite similar to the way the problem is formulated and visually mediated in university (e.g., find q and r such that $7 = q \times 2 + r$); however, seen in a broader context there are significant differences. The shift from arithmetic to algebraic discourse is a central challenge in primary school. Students often experience difficulty even in simple algebraic activities such as addition with an unknown (e.g., $3+? = 8$). The Mathematics Learning Study Committee, in "Adding It Up" (2001), listed some of the perspectives of traditional elementary school arithmetic that need to change in the shift from arithmetic to algebra: "An orientation to execute operations" should shift to an orientation to represent relationships, and "[the] use of the equal sign to announce a result" should shift to signifying an equality (ibid., p. 270). In the school notation of DWR, the equals sign can be seen as announcing the result of a calculation. The university formulation of DWR belongs to an algebraic discourse, where the routine is one of manipulating the right side of the equation $(q \times 2 + r)$ to make it equal to the left side (7). Here, the equals sign represents an equality relationship, not a call to execute a calculation. As such, it belongs to a more advanced-algebraic-discourse.

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Texto 2

USE OF MOBILE TECHNOLOGIES IN MATHEMATICS TEACHING AND LEARNING

BORBA, M.C.; ASKAR, P.; ENGELBRECHT, J.;
GADANIDIS, G.; LLINARES, S.; AGUILAR, M.S.

Several studies have focused on exploiting the capabilities of mobile technologies, such as portability, mobility, and the capacity to take photos and videos of real phenomena that later can be analyzed and discussed from a mathematical point of view. An example is the work of Wijers et al. (2010), who used a location-based game called MobileMath for mobile phones with GPS to allow students to create and explore quadrilaterals and their properties on a real playing field outside the classroom. The researchers found that certain features of this game (like the fact that the game in itself as a whole is competitive) result in quite an engaging experience for the students; in addition, the game allows students to notice geometrical aspects of the real world. However, the authors are cautious and warn that the collected evidence does not allow them to assure that the game has an effect on students' learning. Another example is the study of Daher and Baya'a (2012) in which mathematical applications for mobile phones are used in combination with videos and photos to allow students to perform mathematical analyses of natural phenomena that may occur outside the classroom: for example, measuring the height of lighted candle in intervals of time, registering the results, assigning points in the coordinate system, and using an app for mobile phones (called Fit2Go) to fit a linear or a quadratic function to the assigned points. The authors of this research claim that mobile phones provide students with rich and diverse learning modes (learning through formal manipulations, learning through classroom discussion, learning in an authentic environment), where learning takes place inside and outside the classroom.

Some studies have focused on studying the perceptions and emotions that mathematics teachers and students experience when they teach or study mathematics by using mobile devices. For example, Holubz (2015) studied the perceptions of students and teachers about an initiative called "Bring Your Own Device" (BYOD), where the use of the internet and mobile devices for the study of mathematics is promoted. She investigates how essential the participants of the initiative consider a mobile device to be in the learning of mathematics. In another study, Daher (2011) focuses on examining middle school students' emotions during indoor and outdoor activities while using their mobile phones to learn mathematics. The outdoor activities involved exploring the mathematics of real life phenomena, such as finding the relationship between the circumference of the trunk of a tree and the circumference of its biggest branch. Indoor activities in the classroom included the students discussing graphic and algebraic aspects of the phenomena that they observed and registered outside the classroom. The author of this research concludes that this type of activity promotes positive emotions in students towards the study of mathematics, such as enjoyment, comfort, pleasure, enthusiasm, interest, feeling of time passing, and curiosity; however, admittedly, some students involved in the study experienced the opposite of these emotions.

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Responda às questões 1 a 3 a seguir, com base no texto 1 dado.

Questão 1. Quais são os modelos mais comuns de divisão apresentados no Primeiro Ano? Descreva-os e exemplifique-os.

Questão 2. Qual é o desafio central do ensino de Matemática na Escola Primária? Que sugestões o texto apresenta para supera-lo?

Questão 3. Elabore uma versão em português para o terceiro parágrafo.

Responda às questões 4 e 5 a seguir, com base no texto 2 dado.

Questão 4. Quais as potencialidades de exploração matemática do jogo apresentado? A que conclusão os autores que pesquisaram sua utilização chegaram em relação a sua eficácia para o aprendizado?

Questão 5. Dê exemplos, a partir do texto, de uso do telefone celular para a aprendizagem matemática dentro e fora da sala de aula.